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Modern Algebra and the Rise of Mathematical Structures. By Leo Corry. Basel (Birkhäuser). 1996. 460 pp.

Reviewed by Israel Kleiner

Department of Mathematics and Statistics, York University, Canada

For about three millennia, until the early 19th century, "algebra" meant solving polynomial equations, mainly of degree 4 or less. This has come to be known as *classical algebra*. In the early decades of the 20th century algebra became the study of abstract, axiomatic systems such as groups, rings, and fields. Following the 1930 publication of van der Waerden's *Moderne Algebra*, the subject was called *modern* (or *abstract*) *algebra*.

The transition from classical to modern algebra occurred mainly in the 19th century. In fact, many of the concepts, methods, and results of modern algebra were introduced during this period. What, then, characterizes "modern" or "structural" algebra, embodied in van der Waerden's book, and how does it differ from the algebra of the 19th century? This is the question which Leo Corry addresses in the five chapters comprising Part One of the book under review. He does this by focusing on the development of *ideal theory* from R. Dedekind to E. Noether in the period 1860 to 1930. Part Two of Corry's book, which has four chapters, deals with several formal theories of mathematical structures introduced starting in the 1930s, namely Ore's lattice theory, Bourbaki's theory of "structures," and Eilenberg and Mac Lane's category theory.

Various works in the 19th century can be said to have exhibited algebraic structure; for example, Gauss' theory of binary quadratic forms (1801), Galois' theory of equations (1830), Jordan's *Traité des substitutions et des equations algébriques* (1870), Dedekind's theory of ideals (1871), Frobenius and Stickelberger's work on finite abelian groups containing a proof of the Basis Theorem (1879), and Cartan's structure theorems for Lie algebras (1880s) and associative algebras (1890s). But none of these would be considered by Corry as depicting the structural approach to algebra. In any case, he is not looking for *examples* but rather for a broad definition describing when algebra *as a whole* can be said to have become "structural" or "modern."

Van der Waerden claimed that modern algebra began with Galois; Bourbaki was of the opinion that it originated in Steinitz's 1910 paper on fields. According to Corry, the subject got under way only following the works of Noether (and others) in the 1920s, and was made known to the wider mathematical public by van der Waerden in 1930, in his *Moderne Algebra*.

The author identifies two major elements in any scientific discipline (say algebra), what he calls the “body of knowledge” and the “images of knowledge.” The former deals with the subject matter of the discipline, its content and methods, and includes such things as results, theories, proofs, and open problems. The latter relates to perceptions of the subject by its practitioners, inferred from their works and often reflected in textbooks, and deals with (among other things) the kinds of questions which most urgently need attention, the acceptable types of arguments, and the objectives of research in the discipline. The structural approach in algebra focuses on both the body and the images of the subject and on their interrelations.

Even without a characterization of structural algebra, there would probably be no dissent from the view that van der Waerden’s *Moderne Algebra is* a prime example of that genre. Corry extracts what he sees as the essential structural elements in that book, especially those that differ from the algebra of the 19th century, and posits them as describing “*precisely*” what makes algebra structural (p. 54). Among these elements, relating to both the body and the images of algebraic knowledge, are:

(a) Isomorphism. The recognition that if two algebraic systems are isomorphic, they are mathematically indistinguishable, is fundamental.

(b) The recession of the real and complex numbers into the background; these had been at the root of the algebra of the 19th century.

(c) The relegation to a subsidiary role of the study of solvability of equations (but not of Galois theory), a subject which was at the heart of 19th-century algebra.

(d) The focus on questions of “factorization” in the diverse algebraic systems studied and the concomitant issue of the “building blocks” in such factorizations. For example, the cyclic groups of prime-power order are the building blocks of finite abelian groups.

(e) The study of whether properties of a given algebraic system are inherited by its subsystems, quotient systems, and extensions. For example, if a commutative ring satisfies the ascending chain condition, is the same true of the ring of polynomials over that ring?

(f) Most importantly, the axiomatic method. This is a necessary (though not sufficient) ingredient of modern algebra. In particular, the method must be applied to *all* algebraic systems under consideration (groups, rings, fields, . . .). The axiomatic method, as well as items (a)–(e) above (except for (c)), form a common thread binding the various algebraic systems.

It is important to point out that this is a *retrospective* view of the nature of modern algebra. Neither Noether nor van der Waerden wrote about what modern algebra is or should be.

Part One of Corry’s book focuses on several salient algebraic works of the second half of the 19th century and shows how they fall short on one or another of the above criteria, as well as how they contribute to the rise of various aspects of modern algebra. It concludes with a discussion of the two seminal papers of Noether in the 1920s on ideal theory and contrasts these with related works on the subject, principally by Dedekind but also by Hilbert, Lasker, and Macaulay. We will now comment on aspects of all this. Our brief discussion cannot, of course, do justice to the details or subtleties of Corry’s arguments, but we hope it will convey their sense.

Textbooks are often a good way of divining the images of knowledge in a given period. In Chapter 1 Corry calls special attention to two books on algebra which are representative of their respective periods: Weber's *Lehrbuch der Algebra* of 1895, and (of course) van der Waerden's *Moderne Algebra* of 1930. Algebra was well advanced when Weber's book appeared; there were a number of fundamental structure theorems for diverse algebraic systems (e.g., groups, commutative rings, Lie algebras), though in concrete settings related to the real or complex numbers. In 1893 Weber wrote an important article in which he gave abstract definitions of groups and fields. His goal was to present a formulation of Galois theory as a relationship between groups and fields rather than as a device for solving polynomial equations. This paper, however, had little influence on Weber's contemporaries, and the advanced ideas in the article were, for the most part, either not included or deemphasized in his book, written only two years later. That book represented the prevailing attitudes about what algebra was: the subject dealing with solvability of equations over the real or complex numbers. This is, in fact, the description of algebra that appears in Weber's book.

As late as 1926 the prominent algebraist Hasse still asserted that algebra is the theory of equations; but van der Waerden's *Moderne Algebra* represented a sea change in the conception of algebra. It was a visionary work, incorporating the most recent advances in abstract algebra—by Noether, Artin, and others. It pointed to the future of its subject rather than reflecting its past. There are no more striking testimonials to its impact and to the images of algebra in 1930 than those expressed by Dieudonné and Birkhoff, in 1970 and 1973 respectively:

> I was working on my thesis at the time; it was 1930 and I was in Berlin. I still remember the day that van der Waerden came out on sale. My ignorance in algebra was such that nowadays I would be refused admittance to a university. I rushed to those volumes and was stupefied to see the new world which opened up before me. At that time my knowledge of algebra went no further than *mathématiques spéciales*, determinants, and a little on the solvability of equations and unicursal curves. I had graduated from the École Normale and I did not know what an ideal was, and only just knew what a group was! This gives you an idea of what a young French mathematician knew in 1930. [Dieudonné 1970, p. 137]
>
> Even in 1929, its concepts and methods [i.e., those of "modern algebra"] were still considered to have marginal interest as compared with those of analysis in most universities, including Harvard. By exhibiting their mathematical and philosophical unity and by showing their power as developed by Emmy Noether and her other younger colleagues (most notably E. Artin, R. Brauer, and H. Hasse), van der Waerden made "modern algebra" suddenly seem central in mathematics. It is not too much to say that the freshness and enthusiasm of his exposition electrified the mathematical world—especially mathematicians under 30 like myself. [Birkhoff 1973, p. 771]

Chapter 2 deals with Dedekind's work on ideal theory. This was motivated by his desire to extend Kummer's researches on ideal numbers and unique factorization in cyclotomic integers to more general domains. To that end Dedekind introduced such fundamental algebraic concepts as ring, ideal, field, and module, although in the concrete setting of the complex numbers. For example, a field was defined as a subset of the complex numbers closed under the four algebraic operations.

Dedekind laid considerable stress on the *methods* by which he obtained his results. For instance, in one version of his ideal theory he rejected his earlier proof of a theorem characterizing algebraic integers which had used determinants because they "were alien to the proper content of the theorem" (p. 115). According to Edwards [1992, p. 349], "his insistence on philosophical principles was responsible for many of his important innovations."

His work was conceptual, focusing on the underlying ideas. He introduced some of the basic notions of abstract algebra and proved some of its major results. Among his significant methodological departures were the introduction of the axiomatic method *in algebra* and the institution of set-theoretic modes of thinking. Why, then, is Dedekind not considered the founder of modern algebra? Very briefly, because his use of the axiomatic method (such as it was) was not central to his concerns, and the notion of an axiomatic structure was not part of his images of knowledge. As Corry puts it (p. 70): "Dedekind himself neither advanced a general idea of algebraic structures, nor saw those individual concepts like ideals, modules, fields, groups, etc., as particular instances of a more comprehensive, common conceptual species of 'algebraic structures,' giving rise to similar questions, eliciting similar answers, and deserving a unified formulation and treatment."

The evolution of abstract algebra from Dedekind to Noether, natural in retrospect, was "slow and convoluted" (p. 184). Among its most notable contributors during this period, in terms of both the knowledge and the images of abstract algebra, were Hilbert, Steinitz, and Fraenkel. Their works are discussed in Chaps. 3 and 4.

Hilbert's contributions were in invariant theory, in particular his nonconstructive proof of the so-called Hilbert Basis Theorem for polynomial rings (in which the ascending chain condition appears), in algebraic number theory (his famous *Zahlbericht*), where he extended some of Dedekind's ideas (here he coined the term "ring"), and in his general promotion of the axiomatic method in mathematics. Hilbert did not, however, see ideals—central in invariant theory and algebraic number theory—as a unifying notion, nor did he advance the view that algebraic systems ought to be studied axiomatically. These were to be two central tenets of the structural approach to algebra.

In 1910 Steinitz wrote a groundbreaking 150-page paper, "Algebraische Theorie der Körper," in which he initiated the abstract study of fields as an independent subject. While Weber *defined* fields abstractly, Steinitz *studied* them abstractly. He set for himself no less a task than the determination of *all* fields, starting from the field axioms, and, in an important sense he succeeded. His work was very influential in the development of abstract algebra in the 1920s and 1930s. Van der Waerden claimed that "Steinitz's paper was the basis for all [algebraic] investigations in the school of Emmy Noether" [Van der Waerden 1966, p. 162]. The paper greatly enhanced both the body and the images of algebraic knowledge. It is a brilliant specimen of modern algebra, but only a specimen. Before algebra could be said to have become structural, Steinitz's ideas on fields had to be extended to other algebraic systems.

One such extension—to rings—was attempted by Fraenkel in a 1914 article, "Über die teiler der Null und die Zerlegung von Ringen." His aim was to do for rings what Steinitz had done for fields, namely to give an abstract and comprehensive theory. Here appears the first abstract definition of a ring (though there are two extraneous axioms), and several basic properties of rings are derived. The goal of the paper was to prove a decomposition theorem for rings as direct products of "simple" rings (not the usual notion of simplicity). Fraenkel set for himself too ambitious a task—to subsume both commutative and noncommutative rings under a single theory. Moreover, his theory was not grounded in the existing concrete contemporary theories of rings, and hence it had little direct impact. It did, however, increase to some extent the body of abstract algebraic knowledge, and especially its images: it now became acceptable to study rings as abstract, independent objects, not just as rings of polynomials, as rings of algebraic integers, or as rings of hypercomplex systems (noncommutative rings).

A final noteworthy development, which inspired Noether's studies and increased the body, though not the images, of abstract algebra, was the work of Lasker (in 1905) and Macaulay (in 1913) on polynomial rings. They proved a decomposition theorem stating that every ideal in such a ring is a finite, unique (in some sense) intersection of "primary" ideals.

And now to Noether's work, treated in Chapter 5. In two fundamental papers on ideal theory, in 1921 and 1926 respectively, she began to "change the face of algebra" (Weyl 1981, p. 128]. Mac Lane claimed that "abstract algebra, as a conscious discipline, starts with Emmy Noether's 1921 paper 'Ideal Theory in Ring Domains' " Van der [Mac Lane 1981, p. 10]. Van der Waerden described the essence of Noether's credo [See Birkhoff 1976, p. 42]:

> All relations between numbers, functions and operations become perspicuous, capable of generalization, and truly fruitful after being detached from specific examples, and traced back to conceptual connections.

Specifically, in the 1921 and 1926 papers she extended the work of Hilbert, Lasker, and Macaulay on polynomial rings and of Dedekind on rings of integers of algebraic number fields, respectively, to arbitrary commutative rings. Not only did she thus introduce new concepts and derive new results, she originated "a new and epoch-making style of thinking in algebra" [Weyl 1981, p. 130]. Corry gives a detailed analysis of these contributions—to both the body and the images of algebra. In particular, he compares them to those of her predecessors, mentioned above.

Now some general comments on Part One. The author's *conclusion* that the structural approach in algebra came into being with the works of Noether and her colleagues in the 1920s is eminently reasonable even if one hasn't seen a precise definition of structural algebra. Of course, a "proof" of such a claim depends on the criteria one picks for describing the structural approach. A good case can be made that abstract algebra originated in the 1910s rather than the 1920s if the eminently reasonable criterion is the study of algebraic systems—groups, rings, fields—axiomatically. Thus, as we have seen, Steinitz wrote on abstract fields in 1910 and Fraenkel on abstract rings in 1914. In 1917 and 1918 Sono wrote several papers on rings in a very modern spirit, discussing such structural notion as cosets, quotient rings, maximal and minimal ideals, simple rings, isomorphism theorems, and composition series [Sono 1917–1918]. Groups were studied abstractly as far back as the 1880s; de Séguier wrote a book in 1904 entitled *Eléments de la théorie des groupes abstraits*, and Schmidt another in 1916 on *Abstract Group Theory* [see Wussing 1984]. In 1907 Wedderburn proved his famous structure theorem on finite-dimensional associative algebras *over an arbitrary field* [Wedderburn 1907].

I suspect Corry might agree that the works of Steinitz, Sono, Schmidt, and Wedderburn qualify as structural even in *his* sense. He would no doubt also agree that *his* criteria are not definitive. They do, however, form a useful framework within which to initiate a discussion and analysis of the issues, which he has done with considerable skill. Thus the main value of Part One of the book (to this reviewer, anyway) is not in its thesis, nor even in the criteria set out to establish it, but rather in the airing of the author's thoughtful views in attempting to arrive at the thesis. These are based on a rich panorama which he lays out on aspects of the history of algebra in the 19th and early 20th centuries.

Another point of possible debate is the author's choice of *ideal theory* as the focus of the evolution towards the structural approach in algebra. He gives reasons for the choice (pp. 16–18), and, after all, one cannot range over a broad spectrum of algebraic theories, so one must make choices. Ideal theory is especially well suited to establish the author's thesis: there is a clear, evolving progression of ideas from Dedekind to Noether. It seems to me, however, that picking a subject in group theory or field theory might have made the task considerably more challenging.

Mathematicians rarely reflect on their work—its broad goals in the general scheme of things and the relative importance of the concepts they introduce and the results they prove. But in attempting to elucidate the structural aspects of mathematicians' works, especially their images of the subject, Corry must (and does) deal with such issues. Here are a number of points on which I'd like to question his interpretations:

(a) He argues that for Dedekind, in particular in his work on Galois theory, fields are the *subject matter* of algebra, groups its *tools* (pp. 79, 129), so that fields and groups are for Dedekind "mathematical entities belonging to different conceptual spheres" (p. 79). Since in modern algebra all algebraic systems are to have the same status, Corry alleges that Dedekind's approach was not structural (he also gives other reasons). I doubt that Dedekind made such distinctions between groups and fields. In a similar vein, Corry states that "ideals never appear in Dedekind's work as a special kind of substructure of the more general algebraic structure of a ring" (p. 130), hence ideals and rings are on a different conceptual level. The hypothesis may be correct, but I doubt that it implies the conclusion.

(b) Group theory (as such) was not dealt with by Noether, as Corry correctly points out. He then states that "group theory . . . appeared in van der Waerden's book for the first time as an algebraic theory of parallel status to field theory, ring theory, etc" (p. 252). I do not think this is the case. In fact, among the algebraic subjects group theory was the first to acquire independent and mature status—already in the 19th century, but certainly in the first two decades of the 20th (cf. the books by de Séguier and Schmidt mentioned above; see also [Wussing 1984]).

(c) In speaking of the copious body of algebraic knowledge produced in the period 1860–1930, the author claims that "this growth in the body of knowledge raised many pressing questions concerning the very nature of algebra" (p. 8). I am not aware that such questions were raised at the time; perhaps they *ought* to have been raised.

(d) Speaking of "the gradual adoption [in the mid-1940s] of the structural image in various central mathematical disciplines" (p. 9), Corry states that "under this new conception the idea soon arose that mathematical structures are the actual subject matter of mathematical knowledge in general" (p. 10). I believe this overstates the case. Though this may have been the view of *some* mathematicians, most saw the abstract structures as an *aid*, albeit a very important one, in helping them solve fundamental *problems*.

Part Two of Corry's book, "Structures in the Body of Mathematics," has four chapters and comprises about a third of the book. It describes three theories—what the author calls *reflexive theories*—"which were formulated [in attempts] to elucidate, in strict mathematical terms, the idea of a mathematical structure and its significance within the whole building of mathematics" (p. 255). They are Ore's theory of lattices Bourbaki's theory of "structures," and Eilenberg and Mac Lane's theory of categories. "A detailed analysis of these three theories, their motivations, early evolution and further elaboration or lack of it constitute the main subject of this second part of this book" (pp. 256–257). The structural approach in algebra and in other parts of mathematics raised, according to Corry, new questions about the nature of mathematical knowledge and the role of structures in it, which motivated to a large extent the creation of these (and other) reflexive theories (e.g., Hilbert's metamathematics).

Ore and Birkhoff began to develop lattice theory in the 1930s (Dedekind and Schröder introduced lattices in the late 19th century). Corry focuses on Ore (Chapter 6), who called lattices "structures," since his objective was "to develop a foundation for all of abstract algebra based on the notion of lattice" (p. 263). The idea in analyzing an algebraic system was, according to Ore, not to focus on its elements but rather to focus on "certain *distinguished subdomains*" (p. 272). For example, in two articles in 1937 and 1938, respectively, he attempted, as he put it, "to provide a foundation of the theory of groups, as far as it is possible, directly upon their subgroups, and overlooking their elements" (p. 280). Although Ore's overall programme did not prove fruitful and was soon abandoned, the notion that considerable information about an algebraic system can be gleaned from studying its subsystems is of course still very useful. And in the past several decades lattice theory has had a revival, and has influenced a number of areas of mathematics, notably combinatorics, logic, and geometry.

Bourbaki, probably more than anyone else, is associated with the notion of mathematical structure, although mathematicians have only a general sense of what he meant by it. But in Chapter 4 of Book I of his *Eléments*, entitled "The Theory of Sets," Bourbaki gives a formal definition of "structure." The definition is rather complex (see Chap. 7, p. 321) and is followed by definitions of equivalent structures, derived structures, richer or poorer structures, and finer or coarser structures. The idea, as expressed in a 1950 article, "The architecture of mathematics," was to organize *all of mathematics* around structures: the "mother structures," which are the algebraic, topological, and ordered structures, and various substructures and cross-fertilizing structures. An alluring picture it must have been to many growing up (mathematically) during this period—though of course it had its detractors. Book II of the *Eléments* is called "Algebra," but "[there are] only feeble connections between algebraic structures and *structures* in Bourbaki's *Algebra*" (p. 328). More generally, although Bourbaki's work had great influence (for better or worse) on mathematicians' views of their subject, his formal notion of structure had little.

The penultimate chapter of the book (Chap. 8) discusses the early stages of the development of category theory. The subject arose in a 1942 paper by Eilenberg and Mac Lane titled "Natural isomorphisms in group theory." The idea was to give formal expression to the notion of "naturality." For example, the isomorphism between a finite-dimensional vector space and its dual is not natural (it depends on the choice of a basis), while that between the vector space and its double dual is. This led to the notion of "natural equivalence," which, in turn, led to that of "functor," and finally "category." (This is the reverse of the logical order of the subject and is, in this sense, analogous to the evolution of linear algebra: first came linear transformations, then linear independence, and finally vector spaces.) Category theory has flourished during the past half century—as a subject in its own right and as a language for various branches of mathematics (especially influential was its impact on algebraic geometry). In 1966 Lawvere attempted to use category theory as a foundation for all of mathematics! All this is recounted in detail in Corry's book (look for Bourbaki's attitude to category theory). Category theory has also illuminated important aspects of algebra, notably homological algebra. It is a worthy reflexive theory.

The final, brief, chapter of the book (Chap. 9) describes "how . . . the developments related to these three reflexive theories [discussed above] concomitantly affected the images of mathematics" (p. 384). The focus is on the images of *algebra*, and much of the space is devoted to the ideas of Mac Lane (not a disinterested observer). The thrust of the chapter is that algebra has evolved since the structural approach came into being ca. 1930, and, overall, in the direction of less rather than more abstraction. Its development has been substantially influenced by its interaction with other branches of mathematics, among them geometry, logic, topology, analysis, combinatorics, number theory, and computer science (cf. such fields as algebraic topology, algebraic geometry, topological algebra, algebraic combinatorics, algebraic number theory, algebraic logic, combinatorial group theory, normed rings, theory of automata, and Banach algebras). What, then, is the essence of algebra, Corry asks; and, following several putative attempts at an answer, he comes to the conclusion succinctly summarized in the following quotation from an article by Tamari: "Algebra is what algebraists do or declare to be Algebra, and algebraists are people doing Algebra or declaring themselves to be algebraists" (p. 402). I concur.

Some additional comments on Part Two of the book:

(a) Corry mentions very briefly universal algebra, model theory, and Boolean algebra as possible candidates for reflexive theories of mathematical structure (p. 283). I think that *universal algebra* rather than Bourbaki's structures should have been given detailed treatment. Bourbaki's structures are not very relevant to the structural approach in algebra (it seems as if they are put up as a straw man to be promptly knocked down), while universal algebra is an ideal algebraic reflexive theory: it deals in a unified way with such structural algebraic concerns as isomorphisms, subalgebras, quotient algebras, direct products, chain conditions, and decomposition theorems (see [Cohn 1981, Grätzer 1968]).

(b) Speaking of the evolving nature of algebra, Corry provides as evidence the various editions of Birkhoff and Mac Lane's *A Survey of Modern Algebra* (first edition, 1941). He also mentions Mac Lane and Birkhoff's *Algebra* of 1967, viewing it as one of the editions of the *Survey*. Of course, it is not. It is a different book. It did try to promote a new view of algebra, with a focus on categories and morphisms, but that view did not catch on, certainly not at the undergraduate level (perhaps it will if groups, rings, and fields are taught in high school). The various editions of the *Survey* remained essentially unaltered. One could do worse than adopt the book (5th edition, 1997) as an undergraduate algebra text.

Time to conclude. Corry's *Modern Algebra and the Rise of Mathematical Structures*, which grew out of his doctoral dissertation, is a rich and interesting book. It is well researched and documented, with copious and very useful footnotes, author and subject indexes, and a thorough bibliography of about 500 items. It is written invitingly, though it contains many linguistic infelicities and has been rather sloppily proofread; if a second printing or edition is contemplated, it would require a thorough editorial job. Until such time, have your library get a copy.

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Desargues en son temps. Edited by Jean Dhombres and Joël Sakarovitch. Paris (Blanchard). 1994. 483 pp.

Reviewed by Alessandra Fiocca

University of Ferrara, Department of Mathematics, via Machiavelli 35, 44100 Ferrara, Italy

Studies on Girard Desargues have enjoyed a revival in the middle of the 20th century, thanks to the important research by René Taton [Taton 1951]. Studies on isolated aspects of Desargues' work, particularly on perspective and conic theory, appeared in the eighties and particularly in the nineties in connection with the projected edition of Desargues' complete works by Jean-Pierre Le Goff, René Taton, and Jean Dhombres.

The book under review, *Desargues en son temps,* is connected with this project, and its main characteristic is to propose a global study of Desargues informed by the latest discoveries, while the collaboration between historians of art, architecture, technology, mathematics, perspective, gnomonics, and philosophy provides it with a wide spectrum of contributions. It contains 29 articles in three languages, organized into four main sections.

The first section situates Girard Desargues in his time, both socially and scientifically. It opens with a remarkable chapter by Damish that presents Desargues as a point of connection between art and science and studies the concrete relationships he established with the artists of his epoch, painters, carvers, architects, or stone-cutters. Taton then sketches out a picture of Parisian scientific and cultural life, pointing out the contradictions in Desargues' character, whose intellectual qualities were recognized by the most important mathematicians of his epoch, but who was strongly opposed when he tried to establish graphic techniques on theoretical bases. Two contributions, by Mesnard and Knobloch, examine more specifically Desargues' interesting and varied influence on Blaise Pascal and on Marin Mersenne, respectively. The contributions by Dhombres and Maltese focus on the decisive role of the study of conic sections in the first third of the 17th century, as a phase of transition from the algebra of proportions to analytic geometry and the function concept. The section